

# Einstein-Yang-Mills-Scalar Black Holes with a potential

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We find Einstein-Yang-Mills (EYM) black hole solutions endowed with massless scalar hair in the presence of a potential  $V(\phi)$  as function of the scalar field  $\phi$ . Choosing  $V(\phi) = \text{constant}$  (or zero) sets the scalar field to vanish leaving us with the EYM black holes. Our class of black hole solution has the feature that asymptotically, it is not flat, not anti-de Sitter (AdS) and not conformal to Lifshitz black holes. The role of the potential  $V(\phi)$  in making double bounces (i.e. both a minimum and maximum radii) on a Domain Wall (DW) universe is highlighted.

## I. INTRODUCTION

There has always been curiosity in obtaining exact solutions in Einstein's theory that contain new sources to encompass the underlying spacetime curvature. This covers sources such as scalar (charged, uncharged), cosmological constant, electromagnetism (linear, non-linear, electric, magnetic), Yang-Mills (YM) fields plus others as well as their combinations. The necessity of enrichment in sources can be traced back to gauge / gravity duality, but may find better justification from the more recent holographic superconductivity analogy. Once spherical symmetry and asymptotic flatness in the spacetime ansatz is assumed severe restrictions on possible solutions are inevitable. The uniqueness theorems and no-hair conjectures are a few of such restrictions that the solutions obtained must comply with.

In a recent study minimally coupled scalar field has been considered in Einstein-Maxwell theory with anti-de Sitter (AdS) asymptotics [1]. In this model beside kinetic Lagrangian terms due to the scalar and Maxwell fields an additional potential  $V(\phi)$  constructed from scalar field has been supplemented. Constancy of such a potential can naturally be interpreted as a cosmological term. The scalar field is assumed to be a logarithmic form and upon this choice the form of the potential  $V(\phi)$  can be determined from the dynamical equations. Among other things the model is shown to admit asymptotic Lifshitz black holes which is of utmost importance as far as Hawking radiation is concerned.

In this paper we add Yang-Mills (YM) field instead of Maxwell to the Einstein-minimally coupled scalar field system and search for the resulting solutions. Our pure magnetic YM field is added through the Wu-Yang ansatz which was introduced before [2, 3], and is known to extend easily to all higher dimensions. Our model of Einstein-Scalar-YM system contains an indispensable potential function  $V(\phi)$  as a function of the scalar field. Following the example of Cadoni et al [1] we assume also our scalar field ansatz to be in a logarithmic form. Also since our aim is not to consider the superconductivity analogy, our scalar field is real. Interestingly our model has neither asymptotic flat nor an asymptotic AdS counterpart. Further, in contrast to the Maxwell case our model fails to produce asymptotically a case conformal to Lifshitz metric. Our choice of the metric ansatz is such that  $-g_{tt} = g^{rr}$ , while the coefficient of the unit angular line element  $d\Omega_d^2$  is an arbitrary function  $R(r)^2$ . The asymptotic flatness condition requires that  $R(r) = r$ , which should be discarded in the present case since it annuls the scalar field through the field equations. We observe also that non-trivial scalar and YM fields must coexist only by virtue of a non-zero potential  $V(\phi)$ . The choice  $V(\phi) = \text{constant}$  (or zero) admits no scalar solution. Overall, our solutions can be interpreted as hairy black holes in an asymptotically non-flat spacetime. By setting the scalar field to zero we recover the EYM black holes obtained before [3]. Our model is exemplified with specific parameters of scalar hair. Next, in  $d + 2$ -dimensional bulk spacetime we define a  $d + 1$ -dimensional Domain-Wall (DW) as a Friedmann-Robertson-Walker (FRW) universe by the proper boundary (junction) conditions given long ago by Darmois and Israel [4]. We establish such a DW universe and explore the conditions under which the FRW universe has double bounces. That is, the radius function of FRW universe will lie in between the minimum and maximum values. We had shown previously the existence of such double bounces in different theories [5, 6].

The paper is organized as follows. Our formalism of EYM system with a scalar field and potential is introduced in Section II. Section III presents exact solution to the system under certain ansatzes with an illustrative example. Domain wall dynamics in our bulk space is introduced in Section IV. Our conclusion appears in Section V.

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## II. THE FORMALISM

Following the formalism given in [1] with the same unit convention (i.e.,  $c = 16\pi G = 1$ ) the  $d + 2$ -dimensional Einstein-Yang-Mills gravity coupled minimally to a scalar field  $\phi$  is given by

$$S = \int d^{d+2}x \sqrt{-g} \left[ \mathcal{R} - 2(\partial\phi)^2 - \mathcal{F} - V(\phi) \right] \quad (1)$$

in which  $\mathcal{R}$  is the Ricci scalar,  $\mathcal{F} = \text{tr} \left( F_{\mu\nu}^{(a)} F^{(a)\mu\nu} \right)$  and  $V(\phi)$  is an arbitrary function of the scalar field  $\phi$ . The YM field 2-form components are given by

$$\mathbf{F}^{(a)} = \frac{1}{2} F_{\mu\nu}^{(a)} dx^\mu \wedge dx^\nu \quad (2)$$

with the internal index  $(a)$  running over the degrees of freedom of the nonabelian YM gauge field. Variation of the action with respect to the metric  $g_{\mu\nu}$  gives the EYM field equations as

$$G_\mu^\nu = T_\mu^{\nu(\mathcal{F})} + T_\mu^{\nu(\phi)} - \frac{1}{2} V(\phi) \delta_\mu^\nu \quad (3)$$

in which

$$T_\mu^{\nu(\mathcal{F})} = 2 \left( \text{tr} \left( F_{\mu\alpha}^{(a)} F^{(a)\nu\alpha} \right) - \frac{1}{4} \mathcal{F} \delta_\mu^\nu \right), T_\mu^{\nu(\phi)} = 2 \left( \partial_\mu \phi \partial^\nu \phi - \frac{1}{2} \partial_\rho \phi \partial^\rho \phi \delta_\mu^\nu \right). \quad (4)$$

Variation of the action with respect to the scalar field  $\phi$  yields

$$\nabla^2 \phi = \frac{1}{4} \frac{dV(\phi)}{d\phi}. \quad (5)$$

The  $SO(d+1)$  gauge group YM potentials are given by [7]

$$\begin{aligned} \mathbf{A}^{(a)} &= \frac{Q}{r^2} C_{(i)(j)}^{(a)} x^i dx^j, Q = \text{YM magnetic charge}, r^2 = \sum_{i=1}^{d+1} x_i^2, \\ 2 &\leq j+1 \leq i \leq d+1, \text{ and } 1 \leq a \leq d(d+1)/2, \\ x_1 &= r \cos \theta_{d-1} \sin \theta_{d-2} \dots \sin \theta_1, x_2 = r \sin \theta_{d-1} \sin \theta_{d-2} \dots \sin \theta_1, \\ x_3 &= r \cos \theta_{d-2} \sin \theta_{d-3} \dots \sin \theta_1, x_4 = r \sin \theta_{d-2} \sin \theta_{d-3} \dots \sin \theta_1, \\ &\dots \\ x_d &= r \cos \theta_1, \end{aligned} \quad (6)$$

in which  $C_{(b)(c)}^{(a)}$  are the non-zero structure constants of the  $\frac{d(d+1)}{2}$ -parameter Lie group  $\mathcal{G}$  [2, 3]. The metric ansatz is spherically symmetric which reads

$$ds^2 = -U(r) dt^2 + \frac{dr^2}{U(r)} + R(r)^2 d\Omega_d^2, \quad (7)$$

with the only unknown functions  $U(r)$  and  $R(r)$  and the solid angle element

$$d\Omega_d^2 = d\theta_1^2 + \sum_{i=2}^d \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2, \quad (8)$$

with

$$0 \leq \theta_d \leq 2\pi, 0 \leq \theta_i \leq \pi, 1 \leq i \leq d-1.$$

Variation of the action with respect to  $\mathbf{A}^{(a)}$  implies the YM equations

$$\mathbf{d}^* \mathbf{F}^{(a)} + \frac{1}{\sigma} C_{(b)(c)}^{(a)} \mathbf{A}^{(b)} \wedge^* \mathbf{F}^{(c)} = 0, \quad (9)$$

in which  $\sigma$  is a coupling constant and  $^*$  means duality. The YM invariant is given by

$$\mathcal{F} = \text{tr} \left( F_{\mu\nu}^{(a)} F^{(a)\mu\nu} \right) = \frac{d(d-1)Q^2}{r^4} \quad (10)$$

and

$$tr \left( F_{t\alpha}^{(a)} F^{(a)t\alpha} \right) = tr \left( F_{r\alpha}^{(a)} F^{(a)r\alpha} \right) = 0, \quad (11)$$

while

$$tr \left( F_{\theta_i\alpha}^{(a)} F^{(a)\theta_i\alpha} \right) = \frac{(d-1)Q^2}{r^4}, \quad (12)$$

which leads us to the closed form of the energy momentum tensor

$$T_{\mu}^{\nu(\mathcal{F})} = -\frac{d(d-1)Q^2}{2R^4} \text{diag} \left[ 1, 1, \frac{(d-4)}{d}, \frac{(d-4)}{d}, \dots, \frac{(d-4)}{d} \right]. \quad (13)$$

The above field equations can be rearranged as

$$\frac{R''}{R} = -\frac{2}{d} (\phi')^2, \quad (UR^d\phi')' = \frac{1}{4}R^d \frac{dV}{d\phi}, \quad (14)$$

$$(UR^d)'' = d(d-1)R^{d-2} + 2R^d \left( T_{\theta_i}^{\theta_i(\mathcal{F})} + \frac{2}{d}T_r^{r(\mathcal{F})} \right) - \frac{d+2}{d}R^dV, \quad (15)$$

$$(R^{d-1}UR')' = (d-1)R^{d-2} + \frac{2}{d}R^dT_r^{r(\mathcal{F})} - \frac{1}{d}R^dV, \quad (16)$$

in which a 'prime' denotes derivative with respect to  $r$ . As it was introduced in Ref. [1] we define new variables

$$F(r) = -\frac{2}{d}(\phi')^2, \quad R = e^{\int Y dr}, \quad u = UR^d, \quad (17)$$

which reduce the field equations into

$$Y' + Y^2 = F(r), \quad (u\phi')' = \frac{1}{4}e^{d\int Y dr} \frac{dV}{d\phi}, \quad (18)$$

$$u'' = d(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr} \left( T_{\theta_i}^{\theta_i(\mathcal{F})} + \frac{2}{d}T_r^{r(\mathcal{F})} \right) - \frac{d+2}{d}e^{d\int Y dr}V, \quad (19)$$

$$(uY)' = (d-1)e^{(d-2)\int Y dr} + \frac{2}{d}e^{d\int Y dr}T_r^{r(\mathcal{F})} - \frac{1}{d}e^{d\int Y dr}V. \quad (20)$$

A combination of the last two equations yields

$$u'' - (d+2)(uY)' = -2(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr} \left( T_{\theta_i}^{\theta_i(\mathcal{F})} - T_r^{r(\mathcal{F})} \right) \quad (21)$$

which is a  $V$ -independent equation and can be integrated once to

$$u' - (d+2)uY = \int \left\{ -2(d-1)e^{(d-2)\int Y dr} + 2e^{d\int Y dr} \left( T_{\theta_i}^{\theta_i(\mathcal{F})} - T_r^{r(\mathcal{F})} \right) \right\} dr + C_1. \quad (22)$$

This is further integrated to obtain

$$u = R^{d+2} \left[ \int \left( \frac{1}{R^{d+2}} \left[ \int \left\{ -2(d-1)R^{d-2} + 2R^d \left( T_{\theta_i}^{\theta_i(\mathcal{F})} - T_r^{r(\mathcal{F})} \right) \right\} dr + C_1 \right] \right) dr + C_2 \right] \quad (23)$$

and consequently the potential reads out from (19) as

$$V = \frac{d^2(d-1)}{(d+2)R^2} + \frac{2d}{(d+2)} \left( T_{\theta_i}^{\theta_i(\mathcal{F})} + \frac{2}{d}T_r^{r(\mathcal{F})} \right) - \frac{d}{(d+2)R^d}u''. \quad (24)$$

### III. EXACT SOLUTIONS

We start with an ansatz for the scalar field  $\phi = \alpha \ln \left( \frac{r}{r_0} \right)$  in which  $r_0$  and  $\alpha$  are two real constants. The  $Y$ -equation then, takes the Riccati form

$$Y' + Y^2 = -\frac{2}{d} \frac{\alpha^2}{r^2}, \quad (25)$$

which admits a solution for  $Y$  given by

$$Y = \frac{A}{r} \quad (26)$$

with

$$\begin{aligned} A^2 - A + \frac{2\alpha^2}{d} &= 0 \\ 0 < A < 1. \end{aligned} \quad (27)$$

Note that  $A = 0, 1$  make the scalar field to vanish, so we exclude them. Knowing  $Y$  we find

$$R = \left( \frac{r}{r_1} \right)^A, \quad (r_1 = \text{cons.}) \quad (28)$$

and therefore

$$u = r^{A(d+2)} \left[ \int \left( \frac{-2(d-1)r^{A(d-2)}r_1^{-A(d-2)} + 4(d-1)Q^2r^{A(d-4)}r_1^{-A(d-4)}}{r^{A(d+2)}} dr + C_1 \right) dr + \frac{C_2}{r_1^{A(d+2)}} \right] \quad (29)$$

which becomes

$$u = \begin{cases} \frac{(d-1)r^{2+A(d-2)}r_1^{-A(d-2)}}{(A(d-2)+1)(2A-1)} - \frac{2(d-1)Q^2r^{A(d-4)+2}r_1^{-A(d-4)}}{(A(d-4)+1)(3A-1)} - \frac{C_1}{A(d+2)-1}r + C_2 \left( \frac{r}{r_1} \right)^{A(d+2)}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2} \\ -\frac{4(d-1)\ln(r)r_1^{-\frac{d-2}{2}}r^{\frac{d+2}{2}}}{d} - \frac{8(d-1)Q^2r^{\frac{d}{2}}r_1^{-\frac{d-4}{2}}}{d-2} - \frac{2C_1}{d}r + C_2 \left( \frac{r}{r_1} \right)^{\frac{(d+2)}{2}}, & A = \frac{1}{2} \\ -\frac{9(d-1)r_1^{-\frac{d+2}{3}}r^{\frac{d+4}{3}}}{(d+1)} + 12Q^2r_1^{-\frac{d+4}{3}}r^{\frac{d+2}{3}}\ln r - \frac{3C_1}{d-1}r + C_2 \left( \frac{r}{r_1} \right)^{\frac{(d+2)}{3}}, & A = \frac{1}{3} \\ -\frac{(d-1)(d+2)^2}{d^2r_1^{\frac{d-2}{d+2}}}r^{\frac{3d+2}{d+2}} + \frac{Q^2(d+2)^2}{d-1}r^{\frac{3d}{d+2}} + C_1r\ln r + C_2\frac{r}{r_1}, & A = \frac{1}{d+2} \end{cases} \quad (30)$$

The potential accordingly takes the form from (24)

$$V = \begin{cases} \frac{r_1^{2A}}{r^{2A}} \frac{d(d-1)(A-1)}{(d+2)(2A-1)} + \frac{r_1^{4A}}{r^{4A}} \frac{(1-A)d(d-1)Q^2}{(3A-1)} - dA[A(d+2)-1]C_2\frac{r^{2(A-1)}}{r_1^{2A}}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2} \\ \frac{d(d-1)Q^2r_1^2}{r^2} - \frac{d^2C_2}{4r_1r} + \frac{(d-1)r_1}{r}(d+2+d\ln r) & A = \frac{1}{2} \\ 2(d-1)d\left(\frac{r_1}{r}\right)^{\frac{2}{3}} - \frac{1}{9}\frac{d(d-1)C_2}{r^{\frac{4}{3}}r_1^{\frac{2}{3}}} - \left(\frac{r_1}{r}\right)^{\frac{4}{3}}\frac{dQ^2(4(d-1)\ln r + 3d+9)}{3} & A = \frac{1}{3} \\ (d^2-1)\left(\frac{r_1}{r}\right)^{\frac{2}{d+2}} - \left(\frac{r_1}{r}\right)^{\frac{4}{d+2}}d(d+1)Q^2 - \frac{r_1^{\frac{d}{d+2}}dC_1}{d+2}r^{-2\frac{d+1}{d+2}} & A = \frac{1}{d+2} \end{cases} \quad (31)$$

Let us note that  $r_1$  is a constant introduced for dimensional reason and without lose of generality we set it as  $r_1 = 1$  in the sequel.

#### A. Asymptotic Functions

In this subsection we give the asymptotic behaviors of the general solution found above. To do so we first rewrite the solution (30) in terms of  $R$  (with  $r_1 = 1$ ),

$$u(R) = \begin{cases} \frac{(d-1)R^{\frac{2+A(d-2)}{A}}}{(A(d-2)+1)(2A-1)} - \frac{2(d-1)Q^2R^{\frac{A(d-4)+2}{A}}}{(A(d-4)+1)(3A-1)} - \frac{C_1}{A(d+2)-1}R^{\frac{1}{A}} + C_2R^{d+2}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2} \\ -\frac{8(d-1)\ln(R)R^{d+2}}{d} - \frac{8(d-1)Q^2R^d}{d-2} - \frac{2C_1}{d}R^2 + C_2R^{d+2}, & A = \frac{1}{2} \\ -\frac{9(d-1)R^{d+4}}{(d+1)} + 36Q^2R^{d+2}\ln R - \frac{3C_1}{d-1}R^3 + C_2R^{d+2}, & A = \frac{1}{3} \\ -\frac{(d-1)(d+2)^2}{d^2}R^{3d+2} + \frac{Q^2(d+2)^2}{d-1}R^{3d} + C_1(d+2)R^{d+2}\ln R + C_2R^{d+2}, & A = \frac{1}{d+2} \end{cases} \quad (32)$$

and

$$V(R) = \begin{cases} \frac{1}{R^2} \frac{d(d-1)(A-1)}{(d+2)(2A-1)} + \frac{1}{R^4} \frac{(1-A)d(d-1)Q^2}{(3A-1)} - A(A(d+2)-1)dC_2R^{\frac{2(A-1)}{A}}, & A \neq \frac{1}{2}, \frac{1}{3}, \frac{1}{d+2} \\ \frac{d(d-1)Q^2}{R^4} - \frac{d^2C_2}{4R^2} + \frac{(d-1)}{R^2}, & A = \frac{1}{2} \\ \frac{2(d-1)d}{R^2} - \frac{1}{9} \frac{d(d-1)C_2}{R^4} - \frac{dQ^2(d+3+\frac{4(d-1)}{9} \ln R)}{R^4}, & A = \frac{1}{3} \\ \frac{(d^2-1)}{R^2} - \frac{d(d+1)Q^2}{R^4} - \frac{dC_1}{(d+2)R^{2(d+1)}}, & A = \frac{1}{d+2} \end{cases}. \quad (33)$$

We also note that in terms of  $R$  the line element (7) reads

$$ds^2 = -U(R)dt^2 + \frac{R^{\frac{2(1-A)}{A}}dR^2}{A^2U(R)} + R^2d\Omega_d^2, \quad (34)$$

in which  $U(R) = \frac{u}{R^d}$ . For  $0 < A < \frac{1}{2}$  while  $R \rightarrow \infty$  the line element becomes

$$ds^2 \simeq \xi^2 R^{\frac{2-2A}{A}} dt^2 - \frac{dR^2}{A^2\xi^2} + R^2d\Omega_d^2 \quad (35)$$

in which  $\xi^2 = -\frac{(d-1)}{(A(d-2)+1)(2A-1)}$  so that  $R$  turns into time-like. For  $\frac{1}{2} < A < 1$ , we obtain asymptotically

$$ds^2 \simeq -C_2R^2dt^2 + \frac{R^{\frac{2(1-2A)}{A}}dR^2}{A^2C_2} + R^2d\Omega_d^2. \quad (36)$$

It is observed that since  $A \neq 1$  asymptotic form ( $R \rightarrow \infty$ ) excludes the case of AdS. For the case when  $A = \frac{1}{2}$  one easily finds

$$ds^2 \simeq \lambda^2 \ln(R)R^2dt^2 - \frac{4dR^2}{\lambda^2 \ln(R)} + R^2d\Omega_d^2 \quad (37)$$

where  $\lambda^2 = \frac{8(d-1)}{d}$ . It is needless to restate that the roles of  $t$  and  $R$  change, i.e.  $R$  becomes a time-like coordinate. The asymptotic metrics (35) and (37) suggest also that we have no Lifshitz [7] asymptotes.

## B. An Illustrative example

In this part we study - with some details - the case in which  $A = \frac{1}{2}$  in five dimensions i.e.  $d = 3$ . The metric function therefore is given by

$$U(r) = -\frac{8r \ln r}{3} - 16Q^2 - \frac{2C_1}{d\sqrt{r}} + C_2r \quad (38)$$

and  $R(r)^2 = r$ . The line element becomes

$$ds^2 = -\left(-\frac{8r \ln r}{3} - 16Q^2 - \frac{2C_1}{3\sqrt{r}} + C_2r\right)dt^2 + \frac{dr^2}{\left(-\frac{8r \ln r}{3} - 16Q^2 - \frac{2C_1}{3\sqrt{r}} + C_2r\right)} + rd\Omega_3^2, \quad (39)$$

which is clearly a non-asymptotically flat metric. The horizon can be found by the equation

$$-\frac{8r_h \ln r_h}{3} - 16Q^2 - \frac{2C_1}{3\sqrt{r_h}} + C_2r_h = 0. \quad (40)$$

whose roots can not be expressed analytically. Also to introduce a mass for the possible black hole solution first of all we set  $C_2 = 0$  and then using the Quasi local mass (QLM) [9] concept leads to

$$ds^2 = -F(R)^2 dt^2 + \frac{dR^2}{G(R)^2} + R^2d\Omega_3^2, \quad (41)$$

in which  $F(R)^2 = U(R)$  and  $G(R)^2 = \frac{U(R)}{4R^2}$  while  $U(R) = -\frac{16R^2 \ln R}{3} - 16Q^2 - \frac{2C_1}{3R}$ . The QLM therefore is given by

$$M_{QLM} = \lim_{R \rightarrow \infty} \frac{3}{2} R^2 F[G_r - G] = -\frac{1}{4} C_1 \quad (42)$$

and as a result the line element takes the form

$$ds^2 = - \left( -\frac{8r \ln r}{3} - 16Q^2 + \frac{8M_{QLM}}{3\sqrt{r}} \right) dt^2 + \frac{dr^2}{\left( -\frac{8r \ln r}{3} - 16Q^2 + \frac{8M_{QLM}}{3\sqrt{r}} \right)} + r d\Omega_3^2. \quad (43)$$

Using the definition of Hawking temperature gives

$$T_H = -\frac{4}{5\pi} \left( 1 + \frac{5Q^2}{r_h} + \frac{5}{2} \ln r_h \right) \quad (44)$$

which in order to have positive temperature for  $r_h < 1$ ,  $1 + \frac{5Q^2}{r_h} + \frac{5}{2} \ln r_h < 0$  must hold.

#### IV. DYNAMICS OF DOMAIN-WALLS

In this section we consider a  $d+2$ -dimensional bulk action supplemented by surface terms [10]

$$S = \int_{\mathcal{M}} d^{d+2}x \sqrt{-g} \left[ \mathcal{R} - 2(\partial\phi)^2 - \mathcal{F} - V(\phi) \right] + \int_{\Sigma} d^{d+1}x \sqrt{-h} \{K\} + \int_{\Sigma} d^{d+1}x \sqrt{-h} \mathcal{L}_{DW}, \quad (45)$$

where the DW Lagrangian will be given by  $\mathcal{L}_{DW} = -\hat{V}(\phi)$  as the induced potential on the DW.  $\{K\}$  stands for the trace of the extrinsic curvature tensor  $K_{ij}$  of DW with the induced metric  $h_{ij}$  ( $h = |g_{ij}|$ ).

Herein  $\Sigma$  is the  $(d+1)$ -dimensional DW in a  $(d+2)$ -dimensional bulk  $\mathcal{M}$  which splits the background bulk into two  $(d+2)$ -dimensional spacetimes  $\mathcal{M}_{\pm}$ . Note that  $\pm$  refer to the normal directions on the DW. The metric we shall work on is given by (7) whose parameters are chosen such that  $\lim_{r \rightarrow \infty} U(r) = \infty$  and for the case of BH  $r_h < r = a$ . For the non-BH case we make the choice  $0 < r = a$ . Let's impose now the following condition

$$-U(a) \left( \frac{dt}{d\tau} \right)^2 + \frac{1}{U(a)} \left( \frac{da}{d\tau} \right)^2 = -1 \quad (46)$$

in which the location of the DW is given by  $r = a(\tau)$ , with the proper time  $\tau$ . Therefore the line element on the DW becomes

$$ds_{dw}^2 = -d\tau^2 + a(\tau)^2 d\Omega_d^2. \quad (47)$$

which is a FRW metric in  $(d+1)$ -dimensions with the radius function  $a(\tau)$ . Imposing the boundary conditions i.e., the Darmois-Israel conditions on the wall [4] leads to

$$- \left( \langle K_i^j \rangle - \langle K \rangle \delta_i^j \right) = S_i^j, \quad (48)$$

where

$$S_{ij} = \frac{1}{\sqrt{-h}} \frac{2\delta}{\delta g^{ij}} \int d^{d+1}x \sqrt{-h} \left( -\hat{V}(\phi) \right), \quad (49)$$

is the surface energy-momentum tensor on the DW. Note that a bracket  $\langle \cdot \rangle$  denotes a jump across  $\Sigma$ . The latter yields

$$S_i^j = -\hat{V}(\phi) \delta_i^j. \quad (50)$$

which after (47) and (49) gives the energy density  $\sigma$  and the surface pressures  $p_{\theta_i}$  for generic metric functions  $f(r)$  and  $R(r)$  with  $r = a(\tau)$ . The overall results are given by

$$\sigma = -S_{\tau}^{\tau} = -\frac{d}{4\pi} \left( \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) \quad (51)$$

$$S_{\theta_i}^{\theta_i} = p_{\theta_i} = \frac{1}{8\pi} \left( \frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} + 2(d-1) \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) \quad (52)$$

in which a dot " ." and prime " ' " means  $\frac{d}{d\tau}$  and  $\frac{d}{da}$ , respectively. The junction conditions, therefore read

$$\frac{d}{4\pi} \left( \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) = -\hat{V}(\phi) \quad (53)$$

$$\frac{1}{8\pi} \left( \frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} + 2(d-1) \sqrt{U(a) + \dot{a}^2} \frac{R'}{R} \right) = -\hat{V}(\phi), \quad (54)$$

which upon using

$$\frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} = \frac{2}{\dot{a}} \frac{d}{d\tau} \left( \sqrt{U(a) + \dot{a}^2} \right) \quad (55)$$

and Eq. (54) casts into

$$\frac{U' + 2\ddot{a}}{\sqrt{U(a) + \dot{a}^2}} = \frac{2}{\dot{a}} \frac{d}{d\tau} \left( -\frac{4\pi}{d} \hat{V}(\phi) \frac{R}{R'} \right) = \frac{d}{da} \left( -\frac{8\pi}{d} \hat{V}(\phi) \frac{R}{R'} \right). \quad (56)$$

Finally we find

$$\frac{d}{da} \left( \hat{V}(\phi) \frac{R}{R'} \right) = \hat{V}(\phi). \quad (57)$$

This equation admits a simple relation between  $R(r)$  and  $\hat{V}(\phi)$  given by

$$R'(r) = \xi \hat{V}(\phi) \quad (58)$$

with  $\xi = \text{constant}$ . Using above with some manipulation we obtain the one-dimensional equation

$$\dot{a}^2 + V_{eff}(a) = 0 \quad (59)$$

where the effective potential is defined by

$$V_{eff}(a) = U(a) - \left( \frac{4\pi R}{d\xi} \right)^2. \quad (60)$$

In order that (59) admits a solution as the radius on the DW universe we must have  $V_{eff}(a) < 0$ , which naturally restricts the probable forms of the potential function  $V(\phi)$ . Further, equations (58) yields

$$\hat{V}(\phi) = \frac{Ar^{A-1}}{\xi}, \quad (61)$$

The following boundary condition for the scalar field (see Eq. (38) in reference [10])

$$\frac{\partial \phi}{\partial R} = -\frac{d}{R} \frac{1}{\hat{V}(\phi)} \frac{\partial \hat{V}(\phi)}{\partial \phi} \quad (62)$$

holds automatically ( after fixing  $\xi = 2A$  for convenience).

## V. CONCLUSION

Einstein-Yang-Mills (EYM) fields, minimally coupled with a massless scalar field  $\phi$  supplemented by a scalar potential  $V(\phi)$  is considered. The EYMS system admits exact, both black hole and non-black hole solutions. Depending on the scalar field the potential  $V(\phi)$  has a large spectrum of possible values with the YM-field. The novelty with

the inclusion of the potential  $V(\phi)$  has significant contributions. Firstly, if the potential  $V(\phi)$  is set to zero the scalar field becomes trivial. That is, the YM field within Einstein's general relativity at least in the assumed ansatz spacetime can't coexist with a minimally coupled scalar field. The non-asymptotically flat and non-asymptotically AdS black hole solutions obtained by our formalism admit scalar hair. The mass of such a black hole is naturally the Quasi Local Mass (QLM) apt for the non-asymptotically flat space times. Secondly, the richness brought in by the additional potential  $V(\phi)$  in the bulk spacetime of dimension  $(d+2)$  is employed in the construction of a Domain Wall (DW) universe as a brane in  $(d+1)$ -dimensions. Specifically, we are interested to construct a DW brane as a FRW universe which admits both a minimum and a maximum bounces. We have shown that this is possible, so that once our world is such a brane it will oscillate between these limits ad infinitum. Finally, since a constant potential amounts to a cosmological term a space-filling uniform scalar field may be the cause of the cosmological constant.

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**Figure Caption:**

Examples of Einstein-Yang-Mills-Scalar (EYMS) solutions with double bounces on the domain wall. For technical reasons we restrict ourselves to  $d = 5$  alone. Fig. 1a is the case of non-black hole, while Fig.1b refers to the case of an extremal black hole. The effective potential  $V_{eff}(a)$  is given in Eq. (61). From Eq. (60) the universe admits only the possibility of  $V_{eff}(a) < 0$ . This gives rise to an oscillatory FRW universe on the DW for such an EYMS system when supplemented by a scalar potential in the Lagrangian.



This figure "Figure1ab.png" is available in "png" format from:

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